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LINEAR AND NON-LINEAR PERSPECTIVES ON THE INSTABILITY OF SHEAR FLOW WITH FINE DUST

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ABSTRACT

This paper brings together theoretical models, numerical and laboratory observations to provide a comprehensive understanding of shear instabilities in flows that contain dust particles. This paper also explores how dust affects shear instability in various contexts, including planetary, industrial, and atmospheric environments.

KEYWORDS: *Shear flow, Instability, Dust particles, Rayleigh Number, Richardson Number.*

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INTRODUCTION

Shear flow refers to a type of fluid motion where a velocity gradient is present between adjacent layers of fluid. Shear flow instabilities occur when a fluid flow, in which different layers of fluid move at different velocities (shear flow), becomes unstable. These instabilities are a key concept in understanding the transition from smooth to chaotic flow. Understanding shear flow instabilities is crucial for predicting and controlling turbulence in various engineering and natural systems, such as aircraft wings, weather patterns, and ocean currents. When fine dust particles are introduced into such flows, they can greatly influence their behaviour by addingdrag forces, and changing the local properties including density and viscosity of the fluid.

Fine dust particles are usually small enough to remain suspended in the fluid but large enough to impact the flow through feedback mechanisms. Their presence can either support or hinder the development of shear instabilities based on particle concentration, size distribution, and how particles interact with the fluid. In atmospheric science, for example, layers with dust can influence cloud formation and the spread of pollutants. In astrophysical disks, dust can affect the formation of planets by causing instabilities that lead to particle clustering. Understanding the link between shear flows and dust behavior is crucial for predicting how multiphase flows act in both natural and engineered systems. This study summarizes the significant works carried out in this area of research.

ANALYTICAL STUDIES

The shear flow concept was first introduced by Euler (1755), and Rayleigh (1880) was the first to analyze the shear instabilities. He proposed that for inviscid flow, instability arises when velocity profile has an inflection point. Utilizing half depth of shear layer as feature length and constant velocity at top of shear layer as characteristic velocity, he obtained unstable modes for the wave-number between 0 and 0.639.

Rayleigh (1913) examined the stability of the homogeneous and incompressible shear flow. He determined that the stability of this flow was dependent upon the horizontal shear and found that the flow would be unstable if the shear vorticity exhibited a local maximum in the region.

Rayleigh (1917) examined fundamental rotational flow of an inviscid fluid exhibiting angular velocity ω(r), where r represents the radial distance from axis of rotation. Through a straightforward physical argument, Rayleigh (1917) established his renowned criterion for stability: an essential and sufficient condition for stability of flow concerning axisymmetric perturbations is that square of circulation either increases or remains constant with an increase in r, i.e $\frac{\partial}{\partial r}\left(2\pi r^{2}\omega\right)^{2} \geq 0$. For the case of subsequent applications, this condition is also expressed as $\frac{1}{r^{3}}\frac{\partial}{\partial r}\left(r^{2}\omega\right)^{2} \geq 0$.

Goldstein (1931) examined stability of a shear layer characterized by linear variations in both velocity and density, while maintaining constant values for these parameters outside the layer. Consequently, he incorporated density gradient into the flow type analyzed by Rayleigh (1880). Solberg (1936) examined stability of an axisymmetric baroclinic vortex.

Charney's (1947) seminal research on baroclinic instability is widely recognized. This investigation, utilized a quasi-geostrophic model that is continuous in vertical dimension. In his seminal study, he examined velocities with a constant vertical shear in a continuously stratified environment. He employed quasi-geostrophic approximation inside β-plane, and Charney’s(1947)research has served as foundation for stability studies of synoptic-scale systems in meteorology.

Kuo (1949) expanded Rayleigh's(1917) findings to zonal currents on a rotating Earth and examined shear flow involving exchange of kinetic energy between fundamental linear flow and sinusoidal disturbances.

Miles(1960) investigated the role of parallel shear flowU(y)in generating surface waves. His work established that the rate of energy transfer to wave with a speed of c is dependent on $U''(y)$.

Stability of heterogeneous non-conducting flows has been investigated by many authors Synge(1938), Miles(1960) and Howard(1961). Howard and Guptha (1962) discussed stability of heterogeneous non-conducting fluid between two fixed cylinders with a radial gravitational force, and effect of magnetic field on liquid.

Rudraiah (1970)examined the stability of a heterogeneous, incompressible, non-viscous fluid exhibiting ideal performance, confined between two fixed coaxial cylinders, subjected to an azimuthal magnetic field and radial gravity force, as well as a magnetic field perpendicular to flow. Throughout the analysis, the flow is assumed to be axi-symmetric. His work includes generalization of Miles(1960) theorem (n = ½), the semi-circle theorem (n = 0), and the generalization of Synge’s(1938) theorem (n=1).

Smith and Davis (1982) determined critical threshold beyond which unstable travelling waves occur, based on Miles(1960) work.The magnetic field-induced stability of rotating baroclinic star was investigated by Raghavachar(1984). He derived the necessary condition for stability using normal mode analysis, assuming small wavelengths.

Agarwal and Jaimala(1990a) examined the hydromagnetic stability of non-viscous compressible fluid in a porous medium, giving rise to Darcy resistance force in the equations of motion.They found that magnetic field stabilizes the system by decreasing the growth rate of unstable modes.

Sumathi and Raghavachar (1993) investigated linear stability of plane parallel shear flow in rotating systems in the presence of long wave disturbances, for a general velocity profile. They have derived analytical equations for the instability characteristics and numerically solved a specific case of hyperbolic tangent profile.

Atul Kumar Goel, Agrawal, and Jaimala (1997) extended Miles(1960) sufficient conditions to an incompressible visco-elastic fluid in porous medium based on Brinkman's model. From these conditions the situation that was known to be stable for J>1/4 and which was partly destabilized in presence of a porous medium (Agarwal and Jaimala (1990b)), is expected to be stabilized due to the viscosity of the fluid.

Nidhi Bansal*et al*(1999) examined the effect of weak magnetic field on the stability of an incompressible, viscoelastic shear flow in a porous medium. It was evident that magnetic field and viscosity stabilize the flow while shear and permeability destabilize the flow.

Anshu Agarwal*et al* (2004) studied the instability of a viscoelastic shear layer in an anisotropic porous medium. That is, modes are oscillatory under the condition $D(\rho DU) \geq 0$ everywhere in flow domain, and existence of oscillatory modes can be established even when $D(\rho DU)<0$, provided the condition $D(\rho DU)<2\rho a^2 U$ holds here in the flow domain.

Naresh Kumar Dua and his collaborators (2009) discussed the spatial instability of shear flow in presence of parallel magnetic field in porous medium, and spectra of stable as well as unstable modes were obtained.

Anshu Agarwal*et al*(2014) examined combined effect of shear as well as thermal buoyancy on stability of Oldroydian fluid saturated in porous medium in presence of weak magnetic field. Comparing the result with that obtained by Agrawal and Jaimala (1990a,b) for shear flow instability of thermo-convective flow of Newtonian fluid via porous medium, which shows that when $R_1<0$, system is stable for all wave numbers undercondition $R_D^{-1} > q/2p$holds and observe that the conditions of stability involve all other important parameters involving contribution due to porous medium which is seen to be stabilizing.

Sumathi *et al* (2018), (2019)investigated Hall current influence on 3D, non-parallel, stratified shear flow of incompressible, perfectly conducting fluid. Nonlinear governing equations are solved under assumption of a homogeneous magnetic field. The analysis considers the fluid to be inviscid, incompressible, and perfectly conducting.

Saffman (1962) introduced a basic mathematical model to study how suspended dust particles affectstability of laminar gas flows. The model included two important factors: the dust concentration and a relaxation time (τ). Saffman (1962) employed this model to examineeffect of dust particles on stability of laminar gas flow, specifically examining how dust impacts critical Reynolds number during transition from laminar to turbulent flow.

Michael (1965) posited that in a steady state, dust co-moves with fluid on every side of vortex sheet, exhibiting uniform yet generally distinct mass concentrations along with relaxation times on either side. Under these settings,dust is found to stabilize system by diminishing growth rates of disturbances over time.

Palaniswamy and Purushotham (1981) examined impact of fine dust particles on stability of parallelstratified shear flows based upon Saffman's(1962) model. He considered 2D stability problem of plane parallel shear flow (U(y),0,0) between two parallel planes $y=y_1$ and $y=y_2$ of inviscid, incompressible, stably stratified fluid laden with consistently distributed fine dust particles of uniform size and shape. He found that the sufficient conditions for stability will remain the same as those originally conjectured by Miles(1960).

Evgenys A Smolov and Sergeiv (1998) examined linear stability of incompressible boundary-layer flow of a dusty gas over semi-infinite flat plate. Particles are presumed to be influenced solely by Stokes drag. The issue is simplified to solving modified Orr-Sommerfeld equation (Saffman(1962)). This is resolved numerically through two methodologies: directly via the orthonormalization technique and through the perturbation technique at minimal particle mass content. Stability properties are computed for both mono and polydisperse particles.

Bagewadi and Gireesha (2003) investigated the geometry of streamlines in 2D steady compressible dusty gas flow and developed mathematical models within Frenet frame field system. Velocities of dust and gas are assessed by assuming pressure gradient to be linear, periodic, and exponential.

Sunil *et al* (2004) conducted a theoretical analysis of influence of dust particles on thermal convection in ferromagnetic fluids assuming the presence of an uniform transverse magnetic field. Heat capacity of dust particles increases heat capacity of the fluid, reducing critical magnetic thermal Rayleigh number. Notion of stability exchange is observed to be valid for a ferromagnetic fluid heated from belowwithout particulate matter.

Aravind *et al*(2009) described stability of flow of dusty gas for Boussinesq fluid and determined bounds for the wave velocity of unstable modes.Jaimala *et al*(2010) examined the effect of fine dust particles on stability of parallel shear flows of stably stratified liquid, based upon Saffman's(1962) model assuming a constant horizontal magnetic field. Also,behavior of oscillatory as well as non-oscillatory modes were discussed.

In addition to discussing time evolution of dust surface density distribution using a stochastic model and deriving an advection-diffusion equation, Shugo Michikoshi *et al*(2012) considered a linearized model of gas disk with dust layers and determined the unstable modes.

Aggarwal and Verma (2016) examined the influence of Hall currents on thermal instability of couple-stress fluids containing dust particles. Subsequently, by linearized stability theory as well as normal modes analysis, dispersion relation is derived. He discovered that existence of Hall current generates oscillatory modes that were absent.

NUMERICAL STUDIES

The scientific literature has placed a great deal of emphasis on stability of parallel shear flow in compressible inviscid fluids against infinitesimal perturbations. Kelvin (1880) assumed that the fluid was homogenous, incompressible, non-viscous, and that it was restricted bya finite space with a specified vorticity. It was found that optimum energy is an essential condition for stability.

Yanai and Tokioka (1969) conducted numerical experiments that simulated meridional motions within an axially symmetric vortex. This experiment involves integration of nonlinear inviscid equations of motion inside a domain constrained by rigid limits above and below. The results align with linear theory; however, horizontal wavelength is highly dependent on numerical grid size.

Blumen (1970) examinedstability of barotropic shear flow $\overline{U} = U \ \tanh\left(y/L\right)$ of stably stratified fluid of3D disturbances by linear analysis. He found that for mid latitude, quasi geostrophic flows with small Rossby number, the system is stable to barocline modes and also found that barotropic mode becomes unstable when Rossby number is greater than 0.682(approximately).

Hazel (1972) studied the stability of stratified shear flow of 2D disturbances proposed by Taylor-Goldstein equation, using the Richardson number, wave number, and complex component of disturbance speed. The contribution here was to measure temporal growth rate for particular profiles. This study has come across a peculiar phenomenon in the region $0 \leq J \leq 0.25$, (J is typical Richardson number), where flow assumes a stationary neutral curve, which represents a stability island in an otherwise unstable environment.

Churilov and Shukhman(1988)investigated the weakly unbalanced disturbances of modest magnitude in stratified shear flow. Shukhman(1991) extended this study by investigating the stability of cylindrical mixing layers at high Reynolds numbers.Orszag and Patera (1983), have published extensive literature reviews on shear flow instability.

Buscalioniand Crespo del Arco (1999) considered numerical and theoretical research of natural convection in inclined cavity. The study considered both 2D and 3D enclosures that are heated in the sides. The Prandtl number was assumed to be 0.25 while the inclination was placed at 80^0 from the vertical. They found a distinct difference in the shear instability of 2D and 3D enclosures. The results matched the previous experimental results arrived at by Skeldon *et al* (1996).

Hendrix and Keppens (2014) investigated the impact of dust on Kelvin-Helmholtz instability (KHI) through numerical hydrodynamic simulations of dust as well as gas. They examined the impact of dust on growth rates of KHI in both 2D and 3D contexts, as well as how KHI redistributes and aggregates dust. Additionally, they explored potential connections between structures observed in 3D KHI and those are identified in molecular clouds.

Anu Nath *et al* (2024) examined instability of dusty simple shear flow with non-uniform distribution of dust particles. Their findings, compared with linear stability analysis utilizing Eulerian model, shows that instability originating from inviscid conditions was also characterised by critical wavelength, beneath which it is not sustained. It was also noted that heightened particle inertia mitigates unstable modes, however intensity of instability escalates with strength of connection among fluid as well as particle phases.

EXPERIMENTAL STUDIES

Researchon shear instabilities through experiments exhibitsa rich history in fluid dynamics and material science. Shear instabilities occur when differential motion (shear) within a fluid or between layers of materials generates disturbances that grow over time, leading to turbulent or irregular flow patterns. Several notable experimental contributions have advanced our understanding of this phenomenon.

Research conducted by Gordon and Daniel (1967) showed that when viscous fluid traverses porous material, tangential stress moves fluid near surface at a velocity U_B, which marginally exceeds Q, fluid velocity within bulk of porous medium.

Taylor (1971) posits that when viscous fluid traverses porous solid, it is often presumed that tangential components of surface velocity arezero. When porous material possesses open structure with substantial pores, external surface stress might induce tangential flow beneath surface.

Claudia and Mario (2011) conducted an experimental investigation of velocity profile of fluid subjected to simple shear over porous media. They carried out experiments to study the velocity profile of a fluid subjected to simple shear flow above a porous surface. The porous layers consist of commercial sandpapers with three distinct grit sizes. By gradually lowering the upper plate, they reconstructed the velocity profile, which was observed to remain linear up to about 250 µm from the interface. Extrapolating this profile to the interface allowed to determine the interfacial velocity as a function of the applied stress. The experimental findings were then compared with theoretical predictions obtained by solving the Brinkmann-extended Darcy law within the porous medium, coupled with the Stokes equations in the free fluid, and enforcing velocity continuity and momentum balance at the interface as proposed by Ochoa-Tapia and Whitaker (1995a,b).

CONCLUSION

The instability of shear flows in presence of fine dust particles represents a complex interplay between fluid dynamics and particulate behaviour. Dust particles influence the onset as well as growth of instabilities by modifying local density, generating additional vorticity, and altering momentum exchange between fluid as well as solid phases. These effects can either enhance or suppress instability depending on particle concentration, size, and coupling strength with the flow. This study emphasizes the significant influence of shear flow instability on the development of turbulence, mixing, and transport in velocity-stratified systems, especially in presence of fine dust particles.

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